ITRI626 - Klastoets 2 / Class test 2 – 28 Augustus / August 2019

**Vraag 1 / *Question 1***

Deur ‘n bewys met inferensiereëls te doen, toon aan dat R ⋁ (Q ⋀ ¬P) ⊨ ¬ ((Q ⋀ R) ⇒ P). Wys al jou stappe en redenasies in detail. / *By doing a proof with inference rules show that R ⋁ (Q ⋀ ¬P) ⊨ ¬ ((Q ⋀ R) ⇒ P). Show all your steps and reasoning in detail.* [10]

Solution

We know that (KB ⊨ β) ⇔ (KB ⋀ ¬β) is unsatisfiable.

Assume KB is R ⋁ (Q ⋀ ¬P) and β is ¬ ((Q ⋀ R) ⇒ P).

Consider KB

∴ R ⋁ (Q ⋀ ¬P)

∴ (R ⋁ Q) ⋀ (R ⋁ ¬P) Distributive law

Consider ¬β

∴ (Q ⋀ R) ⇒ P

∴ (¬ (Q ⋀ R)) ⋁ P Implication elimination

∴ (¬Q ⋁ ¬R) ⋁ P De Morgan

∴ ¬Q ⋁ ¬R ⋁ P Simplification

Thus KB ⋀ ¬β is

R1: (R ⋁ Q)

R2: (R ⋁ ¬P)

R3: ¬Q ⋁ ¬R ⋁ P

Proof by inference rules: (6 x ****)

R1, R2 and R3 does not produce a contradiction (□). (****) Consequently, KB ⊨ β does not hold. (****)

**Vraag 2 / *Question 2***

Gee die vier stappe om enige Proposisielogika uitdrukking om te skakel na konjunkte normaalvorm. / *Give the four steps to convert any Propositional expression into conjunctive normal form.* [8]

Solution

Step 1. Eliminate , replacing with (****)

Step 2. Eliminate , replacing with (****)

Step 3. Conjunctive normal form requires to appear only in literals, so we “move inwards” by repeated application of the following equivalences:

(double negation elimination)

(De Morgan)

(De Morgan) (****)

Step 4. Now we have a sentence containing nested and operators applied to literals. We apply the distributivity law, distributing over wherever possible. (****)

**Vraag 3 / *Question 3***

Skakel die volgende uitdrukking om na konjunkte normaalvorm. Wys al jou stappe en redenasies in detail. / *Convert the following expression to conjunctive normal form. Show all your steps and reasoning in detail.*  [10]

Q ⋁ (R ⋀ (P ⇒ R))

Solution

Q ⋁ (R ⋀ (P ⇒ R))

∴ Q ⋁ (R ⋀ (¬P ⋁ R)) Implication elimination

∴ Q ⋁ [(R ⋀ ¬P) ⋁ (R ⋀ R)] Distributive law

∴ Q ⋁ [(R ⋀ ¬P) ⋁ (R)] Simplification

∴ Q ⋁ [(R ⋁ R) ⋀ (R ⋁ ¬P)] Distributive law

∴ Q ⋁ [R ⋀ (R ⋁ ¬P)] Simplification

∴ (Q ⋁ R) ⋀ (Q ⋁ R ⋁ ¬P) Distributive law

Conversion to conjunctive normal form: (6 x ****). Description of inference rules used: (4 x ****)

**Vraag 4 / *Question 4***

Gee al die stappe van die resolusie algoritme. / *Give all the steps of the resolution algorithm.* [9]

Solution

To show that KB ⊧ α, we show that KB ∧ ¬α is unsatisfiable.

Step 1. Convert KB ∧ ¬α to CNF (****)

Step 2. The resolution rule is repeatedly applied to the resulting clauses (****)

Step 3. One of two things happens (****):

1. There are no new clauses that can be added, in which case KB does not entail α (****)
2. Two clauses resolve to yield the empty clause, in which case KB entails α (****)

Totaal [37] / *Total [37]*